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SOME EFFECTS OF TAILPLANE EFFICIENCY ON SUBMARINE STABILITY AND MANOEUVERING

M. Mackay

Defence R&D Canada

Technical Memorandum

DREA TM 2001-031

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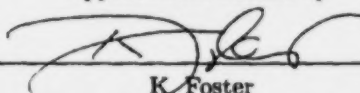
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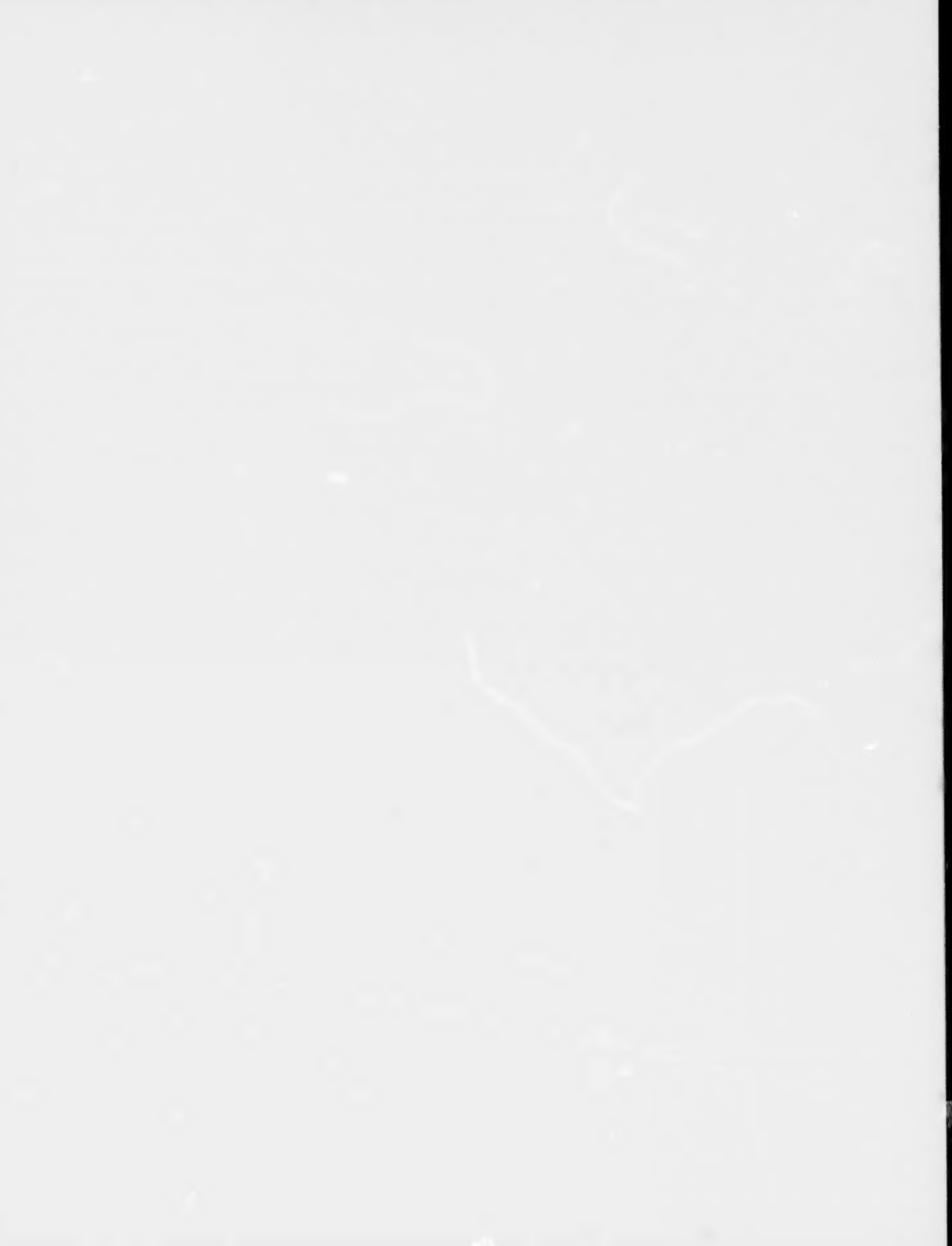
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Abstract

Tailplane efficiency is a measure of the effectiveness of submarine tail appendages working in the hull boundary layer; it can have a significant impact on stability and manoeuvrability. This memorandum describes a study done to observe some of the effects for the DARPA SUBOFF configuration when tailplane efficiency is varied through a range of values that bracket the change from stable to unstable manoeuvring characteristics. The effects are essentially the same as would be obtained by varying the area of the sternplanes and rudders.

Résumé

L'efficacité des gouvernes arrière donne une indication de l'efficacité des appendices arrière d'un sous-marin agissant dans la couche limite de la coque; elle peut avoir une incidence importante sur la stabilité et la manoeuvrabilité. Le présent document décrit une étude réalisée dans le but d'observer certains des effets de la configuration DARPA SUBOFF lorsque l'efficacité des gouvernes arrière varie sur une plage de valeurs qui englobe le passage des caractéristiques de manoeuvre de l'état stable à l'état instable. Les effets sont sensiblement les mêmes que les effets qu'on obtiendrait en faisant varier l'aire des barres de plongée arrière et des gouvernails de direction.

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Executive Summary

Introduction

Tailplane efficiency $k_{\eta t}$ is a measure of the effectiveness of submarine tail appendages working in the hull boundary layer; it can have a significant impact on stability and manoeuvrability. Current prediction methods have to be used with caution because the effects of afterbody geometry and Reynolds number may not be properly accounted for. It is generally expected that $k_{\eta t}$ will be underpredicted for full scale submarines.

This study assesses the impact that this uncertainty in tail efficiency has on the DSSP20 manoeuvring simulation code developed at DREA. The submarine used was the DARPA SUBOFF configuration with hull, sail, and cruciform aft control planes.

Significance

The range of $k_{\eta t}$ used in this study bracketed the extremes of uncertainty associated with current prediction methods. Over this range, small perturbation stability analysis and numerical simulations with DSSP20 demonstrated a change from unstable to stable open loop characteristics that significantly affected manoeuvring and controllability. Improvement in predicting tailplane efficiency is therefore a priority requirement.

Future Development

Model experiments are planned that will provide measurements of tail efficiency for a number of tailplane/afterbody combinations over a useful range of Reynolds numbers. The $k_{\eta t}$ prediction algorithms in DSSP20 will be revised in light of these data.

M. Mackay, 2001, Some Effects of Tailplane Efficiency on Submarine Stability and Manoeuvring, DREA TM 2001-031.

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Sommaire

Introduction

L'efficacité des gouvernes arrière donne une indication de l'efficacité des appendices arrière d'un sous-marin agissant dans la couche limite de la coque; elle peut avoir une incidence importante sur la stabilité et la manoeuvrabilité. Les méthodes de prévision actuelles doivent être utilisées avec prudence, car les effets de la géométrie de l'arrière-corps et du nombre de Reynolds peuvent ne pas être pris en compte adéquatement. On s'attend généralement à ce que ces facteurs soient sous-estimés dans le cas des sous-marins en grandeur réelle.

La présente étude évalue l'incidence de cette incertitude relative aux gouvernes arrière sur le logiciel de simulation de manoeuvre DSSP20 élaboré au CRDA. Le sous-marin utilisé avait la configuration DARPA SUBOFF avec coque, massif, et gouvernes arrière cruciformes.

Signification

La plage d'efficacité des gouvernes arrière utilisée dans l'étude englobait les extrêmes de l'incertitude associée aux méthodes de prévision actuelles. Sur cette plage, une analyse de stabilité en présence de faibles perturbations et des simulations numériques effectuées avec le logiciel DSSP20 ont montré un passage des caractéristiques de boucle ouverte de l'état instable à l'état stable qui influait de façon appréciable sur la manoeuvrabilité et la contrôlabilité. L'amélioration de la prévision de l'efficacité des gouvernes arrière constitue donc une exigence prioritaire.

Développement Futur

On prévoit effectuer des expériences sur modèle qui permettront d'obtenir des mesures de l'efficacité des gouvernes arrière pour un certain nombre de combinaisons de gouvernes arrière/arrière-corps sur une plage utile de nombres de Reynolds. Les algorithmes de prévision du logiciel DSSP20 seront révisés à la lumière des résultats qui seront obtenus.

M. Mackay, 2001, Some Effects of Tailplane Efficiency on Submarine Stability and Manoeuvring, DREA TM 2001-031.

Centre de Recherches pour la Défense Atlantique.

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1 Introduction

Tailplane efficiency $k_{\eta t}$ is the ratio of lift generated by an actual submarine tail appendage or control surface to the lift generated by an equivalent appendage mounted on a reflecting plane in the free stream. It incorporates the principal fin-body interaction effects that arise from a submarine tailplane arrangement, but not the influence of a propulsor.

Empirical methods are available for estimating $k_{\eta t}$, but their generality is open to question, especially with regard to modeling afterbody geometry and Reynolds number effects. There appear to have been no relevant experiments done in which the effect of Reynolds number has been systematically investigated. To remedy this omission, DREA and BWB (German MoD) are co-sponsoring a series of hull/sternplane tests, over a useful range of Reynolds numbers, to take place in late 2001.

To assess the impact that the current uncertainty in predicting tailplane efficiency has on manoeuvres simulated using the DSSP20 code (DREA Submarine Simulation Program, version 2.0), a series of simulations was done for the fully-appendaged generic DARPA SUBOFF geometry with a range of $k_{\eta t}$ specified for the sternplanes and the rudders. The conventional stability indicators were first obtained from small perturbation analysis using the linearized equations of motion. The simulations, which use nonlinear equations of motion, concentrated on combinations of tailplane efficiency and speed that were indicated to have marginal stability.

This memorandum gives some background on the estimation of tailplane efficiency and discusses results from the analysis and DSSP20 simulations of DARPA SUBOFF. The small perturbation analysis method is outlined in annex A.

In calculating the lift, or normal force, for similar tail appendages, a difference in area can be modeled, to first order, by a corresponding difference in $k_{\eta t}$. (Second order effects that are ignored include those due to inevitable differences in aspect ratio and other parameters.) Therefore, the effects of varying $k_{\eta t}$ as described in this memorandum are, to first order, the same as those that would be observed by systematic proportional changes in the rudder or sternplane areas.

2 Predicting Tailplane Efficiency

A widely used empirical method of estimating $k_{\eta t}$ was proposed by Dempsey [1], and is based on model-scale measurements of incremental force and moment for a family of tail appendages mounted on a slender hull. The experiments were done at a hull Reynolds number of 14 million, and appendage chord Reynolds numbers were between about 0.3 and 1.0 million; no turbulence tripping was used. In this method, $k_{\eta t}$ is expressed as a function of b/r_M , the ratio of semispan from the hull centerline to

maximum hull radius

$$k_{\eta t} = -0.3644 + 1.2380 \frac{b}{r_M} - 0.3728 \left(\frac{b}{r_M} \right)^2 \quad \text{for} \quad 0.734 \leq \frac{b}{r_M} \leq 1.426 \quad (1)$$

and

$$k_{\eta t} = 1 - \frac{0.2556}{\left(\frac{b}{r_M} \right)^2} \sqrt{\left(\frac{b}{r_M} \right)^2 - 0.1612} - 0.6366 \sin^{-1} \left(\frac{0.4015}{\frac{b}{r_M}} \right) \quad (2)$$

for $0.4015 < \frac{b}{r_M} < 0.734$ and $1.426 < \frac{b}{r_M} < \infty$

The singularity at $b/r_m = 0.4015$ is below the practical limit for submarine appendages. However, a difficulty with using b/r_m as a parameter is that it does not take account of Reynolds number (so there is no guidance for scaling $k_{\eta t}$ up to full scale), nor does it incorporate the afterbody geometry.

Equations (1) and (2) predict values between about 0.3 and 0.7 for submarine sternplanes and rudders, but it is generally accepted that the method tends to underestimate $k_{\eta t}$ for full scale. There is also evidence that $k_{\eta t}$ may be greater in model tests, even approaching unity at high model Reynolds numbers. Figure 1 illustrates this observation for data from a number of sources including Dempsey [1] (representing low Reynolds number), Lyons and Bisgood [2], and Freeman [3]. Unfortunately, the evidence is limited, and in some cases is marginally representative for submarine configurations.

In the absence of more reliable data, Dempsey's and other prediction methods have to be used with caution. The study described in this memorandum has, as its principle objective, the aim of better understanding the impact that uncertainty in estimating $k_{\eta t}$ has on numerical simulations of submarine manoeuvres.

3 Stability Derivatives

The linear hydrodynamic coefficients, e.g., Y'_v and Z'_w , in the standard submarine equations of motion [4] are often called the stability derivatives. For this description, they must be obtained for vanishingly small perturbations in v , w , etc. (but this may not always have been the case when specific values are cited). Their role in determining the dynamic stability of the submarine is outlined in annex A.

Ignoring interaction effects, it is straightforward to derive first order approximations for the change in stability derivatives in response to a change in tailplane efficiency or area. Here, such changes are written $a \leftarrow a + \dots$, indicating that the quantity a on the left hand side is to be replaced with the expression on the right hand side. For the horizontal plane, in response to $k_{\eta t} \leftarrow (1 + f) k_{\eta t}$ (or to $\text{area} \leftarrow (1 + f) \text{area}$), where f is a fractional increase or decrease

$$\begin{aligned}
Y'_v &\leftarrow Y'_v + \frac{f}{h_r} Y'_{\delta r} \\
N'_v &\leftarrow N'_v + \frac{f}{h_r} N'_{\delta r} \\
Y'_r &\leftarrow Y'_r + \frac{f}{h_r} (x'_{\delta r} + g_r) Y'_{\delta r} \\
N'_r &\leftarrow N'_r + \frac{f}{h_r} (x'_{\delta r} + g_r) N'_{\delta r} \\
Y'_{\delta r} &\leftarrow (1 + f) Y'_{\delta r} \\
N'_{\delta r} &\leftarrow (1 + f) N'_{\delta r}
\end{aligned} \tag{3}$$

Derivatives in the right hand side expressions are before modification. For the rudders, h_r is control effectiveness $dv'/d\delta_r$, $x'_{\delta r}$ is their location, and $(x'_{\delta r} + g_r) = dv'/dr'$, where g_r accounts for yaw-induced camber. Similarly, in the vertical plane

$$\begin{aligned}
Z'_w &\leftarrow Z'_w + \frac{f}{h_s} Z'_{\delta s} \\
M'_w &\leftarrow M'_w + \frac{f}{h_s} M'_{\delta s} \\
Z'_q &\leftarrow Z'_q - \frac{f}{h_s} (x'_{\delta s} + g_s) Z'_{\delta s} \\
M'_q &\leftarrow M'_q - \frac{f}{h_s} (x'_{\delta s} + g_s) M'_{\delta s} \\
Z'_{\delta s} &\leftarrow (1 + f) Z'_{\delta s} \\
M'_{\delta s} &\leftarrow (1 + f) M'_{\delta s}
\end{aligned} \tag{4}$$

where, for the sternplanes, h_s is control effectiveness $dw'/d\delta_s$, $x'_{\delta s}$ is their location and $-(x'_{\delta s} + g_s) = dw'/dq'$, where g_s accounts for pitch-induced camber. Note that h_r is negative, h_s is positive, and that both $x'_{\delta r}$ and $x'_{\delta s}$ are negative with the coordinate with origin near the CG. Both g_r and g_s are negative and small (they are approximately equal to $-\bar{c}/2$, where $\bar{c}$ is the mean appendage chord); neglecting them results in an error of only a few percent.

4 DSSP20

DSSP20 is a suite of programs developed at DREA to simulate the manoeuvres of submarines and other underwater vehicles using a description of external geometry to determine their hydrodynamic characteristics [5,6]. Edition 001214, incorporating recent improvements in drag calculation, was used for the present work.

The default estimation of $k_{\eta t}$ employs Dempsey's method for an appendage chord Reynolds number of less than one million, uses the value 1.0 if it is greater than two

million, and interpolates linearly in between. As previously discussed, there are deficiencies in this at both ends of the Reynolds number range, so provision is made to input a value individually for each appendage. This value is usually arrived at by fine tuning the estimate with suitable test or trials data. For the present study direct input is used to specify values of $k_{\eta t}$ for the rudders and sternplanes.

DSSP20 does not generate the vehicle stability derivatives directly. They are obtained from regression analysis of simulations (with DSSP20B [6]) of a submarine undergoing forced variation of incidence, rotational velocity, or control deflection.

5 DARPA SUBOFF

The submarine used in this study is the DARPA SUBOFF [7] configuration with hull, sail, and cruciform aft control planes. Initial validation of DSSP20 predictions was obtained at model scale (length 4.26085 m) for comparison with Roddy's towing tank data [8]. Roddy's experiments were done at 6.5 kt, giving a hull Reynolds number of 14 million, and a tail appendage chord Reynolds number of about 0.65 million. At this low chord Reynolds number, DSSP20 defaults to using Dempsey's method to estimate $k_{\eta t}$, resulting in a value of 0.5008. With this value, reasonable agreement was found between predictions and data. The submarine was predicted to be unstable in both the horizontal and vertical planes.

For manoeuvring simulations, the submarine was scaled to a length of 91.44 m, with an assumed $\overline{BG}$ of 0.5 m and a top speed of 30 kt. Reference [7] does not define a propulsor, so the simulations use the DSSP20 default Wageningen B series propeller with a propeller/hull diameter ratio of 0.5. At full scale Reynolds numbers, letting DSSP20 use the default $k_{\eta t} = 1$, i.e., doubling the model scale value, results in stability being predicted in both planes. It was this potential discrepancy between predictions of model scale and full scale behavior that prompted the present study.

DSSP20 input files defining this DARPA SUBOFF configuration at full scale are listed in annex B.

6 Horizontal Plane Stability and Manoeuvring

Stability and manoeuvring in the horizontal plane were evaluated for six values of rudder efficiency, from $k_{\eta t} = 0.5$ to 1.0 in steps of 0.1. For the sternplanes, $k_{\eta t}$ was fixed at 1.0 in order to maintain stability for depth control. The estimated horizontal stability derivatives varied nearly linearly with $k_{\eta t}$, in accordance with equations (3); see figures 2, 3, and 4.

The linearized equations of motion (annex A) give, for small perturbations, a horizontal stability index, G_h (equation (A.3)), and a neutral point separation, s'_h

(equation (A.4)). A further indicator of stability is the slope of curvature in a turn, $dr'/d\delta_r$ (equation (A.5)); however, since it goes through a singularity between unstable and stable configurations, the inverse slope is used instead. Conditions for stability are

$$G_h > 0 \quad \text{or} \quad s'_h > 0 \quad \text{or} \quad \frac{d\delta_r}{dr'} < 0 \quad (5)$$

These quantities are plotted against $k_{\eta t}$ in figure 5. They are all equal to zero for $k_{\eta t} = 0.824$; the cases $k_{\eta t} = 0.5, 0.6, 0.7$, and 0.8 are unstable, and the cases $k_{\eta t} = 0.9$ and 1.0 are stable.

It is common to design for marginal stability, or even marginal instability, in the horizontal plane, i.e., so that G_h , s'_h , and $d\delta_r/dr'$ are all small. However, these indicators provide limited insight into actual dynamic behavior, particularly as the assumption of small perturbations becomes less valid.

For each of the six values of rudder $k_{\eta t}$, manoeuvring simulations were done to determine turning characteristics. Stability in the horizontal plane results in the curvature, or nondimensional turning rate, r' , being a monotonic function of rudder angle, δ_r (with, in conventional notation, negative slope). Instability, on the other hand, results in hysteresis in the r' vs. δ_r curve [9], as illustrated in figure 6. In this figure, the arrows indicate how the submarine responds to a continuous variation of rudder angle. For example, r' is negative at a large positive rudder angle, so the submarine turns to port; as rudder angle is reduced, it continues to turn to port but turning rate decreases. The turn to port continues as rudder angle is reduced below zero, to a point (between $\delta_r = -3$ and -4 degrees in this case) at which it abruptly becomes a higher rate turn to starboard. The slope $dr'/d\delta_r$ at zero rudder angle, which is positive, has no physical meaning for the unstable condition.

Two types of turning manoeuvres [9] were simulated:

- 270 degree pullouts. The rudder is deflected from zero and held at a set value until the submarine has changed heading by 270 degrees; the rudder is then returned to zero.
- 270 degree spirals. The rudder is moved from a positive to negative deflection in small increments, holding each incremental value for a change of heading of 270 degrees. Generally, the manoeuvre is completed by returning the rudder to its original deflection in the same increments.

Initial speed was 15 kt and depth was maintained using the sternplanes.

For the unstable cases, a complete spiral was done to delineate the hysteresis region of the r' vs. δ_r curve, and pullouts were done outside it. For the stable cases, a positive-to-negative half spiral over the whole rudder angle range was sufficient.

Figure 7 shows pullout trajectories with rudder angles of 5, 10, and 15 degrees for the most unstable case, $k_{\eta t} = 0.5$. Note that turning rates are very high over the first 270 degrees with the rudder deflected (e.g., at $\delta_r = 15$ degrees, the tactical diameter is less

than three submarine lengths), and that all trajectories continue with a fairly high turn rate afterwards with the rudder returned to zero.

Figure 8, the full set of r' vs. δ_r curves (where r' is the steady value reached after a 270 degree change of heading) summarizes the effect of rudder efficiency on turning characteristics. Both pullouts (open symbols) and spirals (closed symbols) are included. For marginal instability with $k_{\eta t} = 0.7$ to 0.8 , the 'dead-rudder' region, where there is hysteresis in the steering response, is limited to only about ± 1 degree, while sensitivity to slightly larger deflections remains quite large. For marginal stability, $k_{\eta t} = 0.9$ to perhaps 1.0 , there is still a region within $\delta_r = \pm 3$ degrees where the steering response (as measured by $dr'/d\delta_r$) is greater than at larger rudder deflections. There is a slight asymmetry in the turning characteristics that results from the sense of propeller rotation.

7 Vertical Plane Stability and Manoeuvring

A similar procedure was used to evaluate stability and manoeuvring in the vertical plane. Six values of sternplane efficiency were used, from $k_{\eta t} = 0.5$ to 1.0 in steps of 0.1 . For the rudders, $k_{\eta t}$ was fixed at 1.0 in order to maintain stability for heading control. The estimated vertical stability derivatives varied nearly linearly with $k_{\eta t}$, in accordance with equations (4); see figures 9, 10, and 11.

Annex A notes that at low speed the submarine is always stable, provided there is metacentric stability. For the limiting case of high speed, there is a vertical plane stability index, G_v (equation A.8), and a neutral point separation, s'_v (equation A.9), that are equivalent to their horizontal plane counterparts. Satisfying

$$G_v > 0 \quad \text{or} \quad s'_v > 0 \quad (6)$$

is sufficient to guarantee stability at all speeds. These quantities are plotted against $k_{\eta t}$ in figure 12. They are both equal to zero for $k_{\eta t} = 0.834$. If the high speed stability conditions are not met, there is some intermediate critical speed, U_c (equation A.10), at which low speed stability becomes high speed instability. U_c is plotted against $k_{\eta t}$ in figure 13.

These indicators provide limited insight into actual dynamic behavior in the vertical plane. However, some sense of it is provided by the damping ratio, ζ (the ratio of actual to critical damping, equation A.11). Figure 14 shows how ζ varies with speed for each value of $k_{\eta t}$. Note that it becomes zero at U_c for the unstable cases $k_{\eta t} = 0.5, 0.6, 0.7$, and 0.8 . Unlike for the horizontal plane, it is generally considered prudent to design for stability in the vertical plane by satisfying equation (6) by a sufficient margin to achieve a satisfactory damping ratio at high speed (e.g., $\zeta > 0.5$).

For this study, at each of the six values of sternplane $k_{\eta t}$, a series of $10^\circ/10^\circ$ depth change manoeuvres [9] was done at speeds around U_c , i.e., in what is predicted to be the region of marginal stability or instability:

- 10°/10° depth change. From a level straight-ahead trajectory, the sternplanes are deflected 10 degrees (i.e., to dive), held at that value until a 10 degree pitch (down) angle is achieved, and then returned to zero. This is done open-loop; there is no compensation for overshoots.

Figures 15 and 16 show the resulting depth and pitch time histories for $k_{\eta t} = 0.5$. A change from damped stable behavior to amplitude-limited oscillatory behavior (limit cycling [12]) occurs between 9 and 10 kt, consistent with U_c . Mean depth after the manoeuvre increases roughly linearly with speed. Similar results are shown for $k_{\eta t} = 0.6$ and 0.7 in figures 17 to 20. However, as $k_{\eta t}$, and hence U_c , increases, the depth excursion and pitch motion are not damped completely to zero in the region of marginal stability so that limit cycling starts being observed at speeds below U_c . This is illustrated for $k_{\eta t} = 0.8$ in figures 21 and 22, where limit-cycling becomes evident above 20 kt although the U_c estimate is 28.44 kt. The numerical model no longer reflects nonlinear dynamics at these high speeds.

This memorandum has not discussed uncertainty in the predicted stability indicators because that is in large measure dependant on the procedures used to estimate the stability derivatives — in the present case, using regression on nonlinear numerical simulations, with a likely level of uncertainty of between five and ten percent. Figure 13 shows that U_c is very sensitive to estimation uncertainty in the region of the high speed limit, while figure 14 shows that damping for marginal stability is both small and relatively insensitive to speed in the same region. As discussed in the previous paragraph, depth change characteristics for $k_{\eta t} = 0.8$ (and, to a lesser extent, those for $k_{\eta t} = 0.6$ and 0.7) indicate that the change from stable to unstable behavior occurred at a lower speed than U_c . This observation was confirmed for small perturbation manoeuvres in which the sternplanes were deflected to one degree for one second; $k_{\eta t} = 0.8$ pitch time histories for speeds from 21 to 30 kt are shown in figures 23 and 24.

However, the high speed limit remains valid. Stable cases $k_{\eta t} = 0.9$ and 1.0, figures 25 to 28, show a little less than critical damping, as was indicated in figure 14. For safe operation in the vertical plane, the $k_{\eta t} = 1.0$ characteristics in figures 27 and 28 would probably be considered satisfactory. In closed loop simulations a helmsman or autopilot would further reduce or eliminate the overshoots.

Plane reversal speed U_{cs} , equation A.12, was not an issue for the present work since, by modifying the vertical stability derivatives according to equations (4), it is straightforward to show that U_{cs} is independent of variation in $k_{\eta t}$. For all configurations examined here, U_{cs} was equal to 4.31 kt.

8 Concluding Remarks

Classic small perturbation stability analysis and numerical simulations of the DARPA SUBOFF submarine show that within the extremes of uncertainty in predicting tail

efficiency $k_{\eta t}$ there may be a change from unstable to stable open loop characteristics that significantly affects manoeuvring and controllability. This uncertainty is not unique to DSSP20, since it employs a widely-used prediction method that does not account for variations in afterbody geometry or Reynolds number. The present DSSP20 default correction for Reynolds number is based on data that are too few, and are in some cases marginally representative for submarine configurations.

While clearly undesirable from the point of view of analysis and evaluation, underprediction of tail efficiency in the preliminary stages of design will result in somewhat oversized appendages that are conservative from the point of view of stability. Upcoming hull/sternplane tests should throw a valuable light on this problem area, and permit the development of improved algorithms for predicting $k_{\eta t}$.

The results presented for a change in tailplane efficiency apply, to first order, to a corresponding change in tailplane area. So for DARPA SUBOFF, if one accepts the estimate $k_{\eta t} \approx 0.5$ from Dempsey's method (confirmed by Roddy at model scale), then doubling rudder and sternplane areas will result in a configuration that is more-or-less acceptable for stability and control.

References

1. Dempsey, E.M. (1977). Static Stability Characteristics of a Systematic Series of Stern Control Surfaces on a Body of Revolution. (DTNSRDC Report 77-0085). David Taylor Naval Ship Research and Development Center.
2. Lyons, D.J. and Bisgood, P.L. (1945). An Analysis of the Lift Slope of Aerofoils of Small Aspect Ratio, including Fins, with Design Charts for Aerofoils and Control Surfaces. (ARC R&M No. 2308). Aeronautical Research Council.
3. Freeman, H.B. (1932). Force Measurements on a 1/40-Scale Model of the U.S. Airship "Akron". (NACA Report No. 432). National Advisory Committee for Aeronautics.
4. Gertler, M. and Hagen, G.R. (1967). Standard Equations of Motion for Submarine Simulation. (NSRDC Report 2510). Naval Ship Research and Development Center.
5. Mackay, M. (1999). DSSP20 (Beta Edition) User Guide to the Preprocessing Modules. (DREA TM 1999-108). Defence Research Establishment Atlantic.
6. Mackay, M. (1999). DSSP20 (Beta Edition) User Guide to the Simulation Modules. (DREA TM 1999-109). Defence Research Establishment Atlantic.
7. Groves, N.C., Huang, T.T., and Chang, M.S. (1989). Geometric Characteristics of DARPA SUBOFF Models (DTRC Model Nos. 5470 and 5471). (DTRC/SHD-1298-01). David Taylor Research Center.
8. Roddy, R.F. (1990). Investigation of the Stability and Control Characteristics of Several Configurations of the DARPA SUBOFF Model (DTRC Model 5470) from Captive-Model Experiments. (DTRC/SHD-1298-08). David Taylor Research Center.
9. Comstock, J.P. (1967) (editor). *Principles of Naval Architecture*, 1967 edition. New York: SNAME.
10. Abkowitz, M.A. (1972). *Stability and Motion Control of Ocean Vehicles*. Cambridge MA: M.I.T. Press.
11. Burcher, R. and Rydill, L. (1994). *Concepts in Submarine Design*. Cambridge: CUP.
12. Palm, W.J. (1983). *Modeling, Analysis, and Control of Dynamic Systems*. New York: Wiley.

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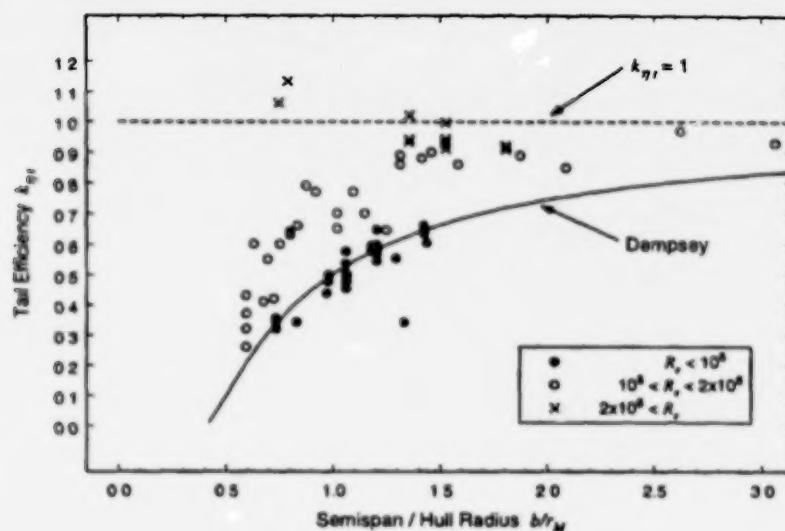


Figure 1. Model scale data for $k_{\eta t}$ at various appendage chord Reynolds numbers R_e

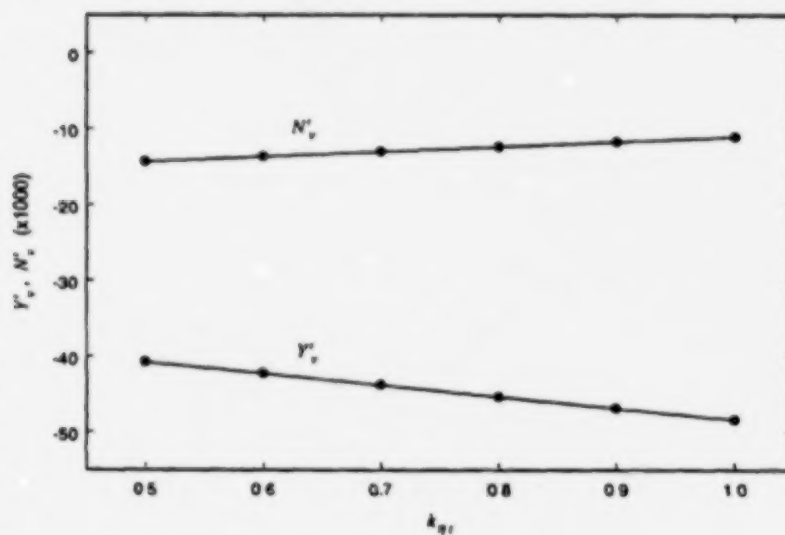


Figure 2. Yaw stability derivatives as functions of $k_{\eta t}$

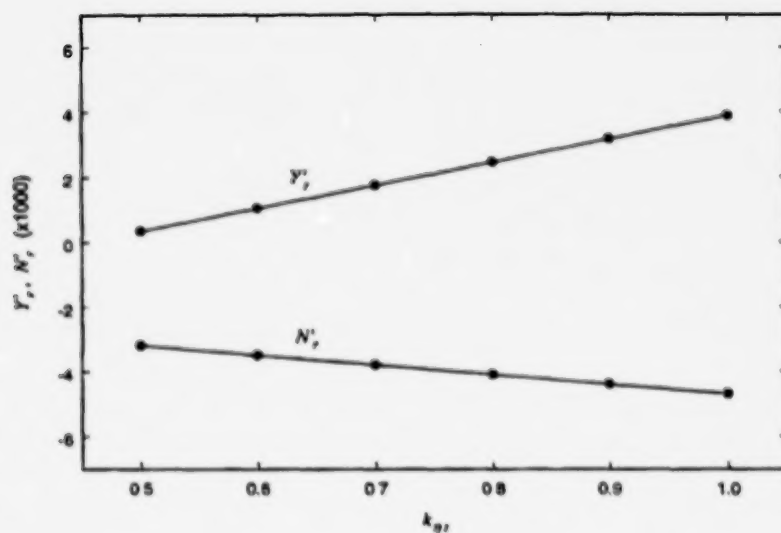


Figure 3. Yaw rate stability derivatives as functions of $k_{\eta t}$

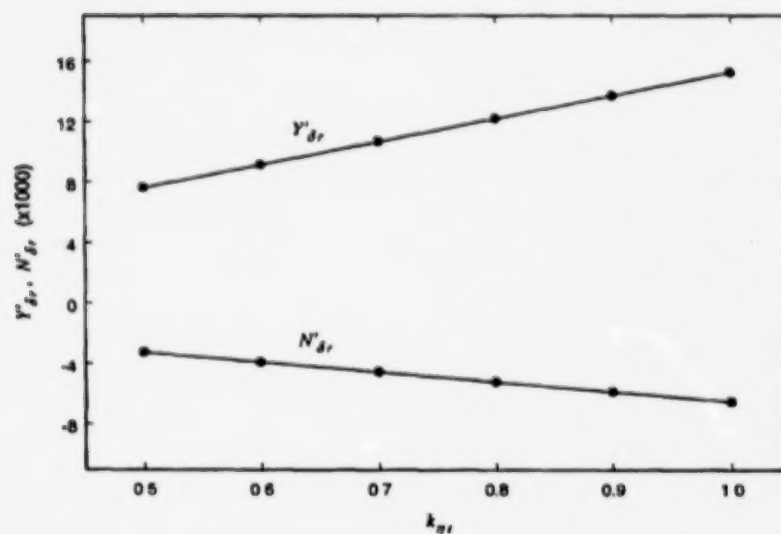


Figure 4. Rudder deflection stability derivatives as functions of $k_{\eta t}$

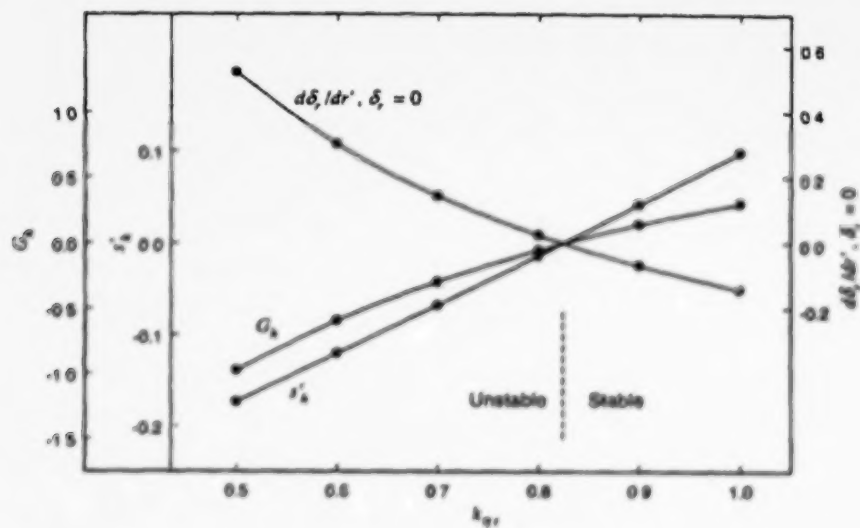


Figure 5. Horizontal plane stability indicators as functions of k_{yr}

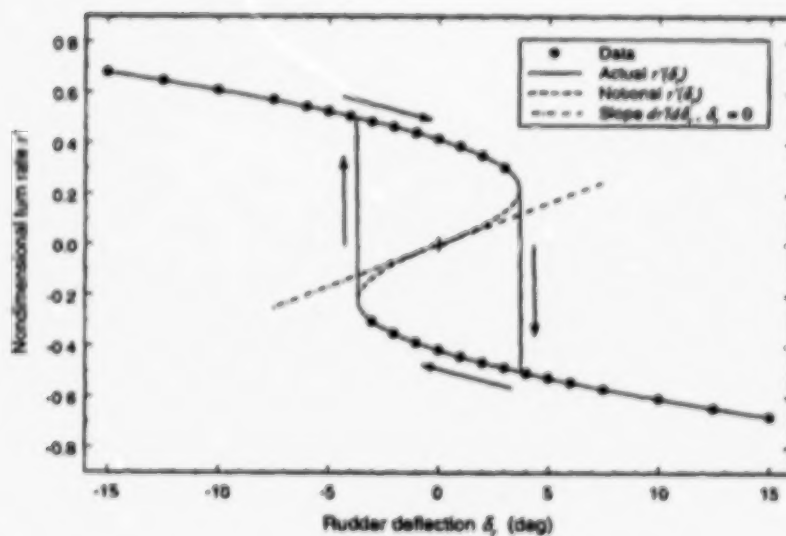


Figure 6. Nondimensional turning rate vs. rudder angle for an unstable submarine

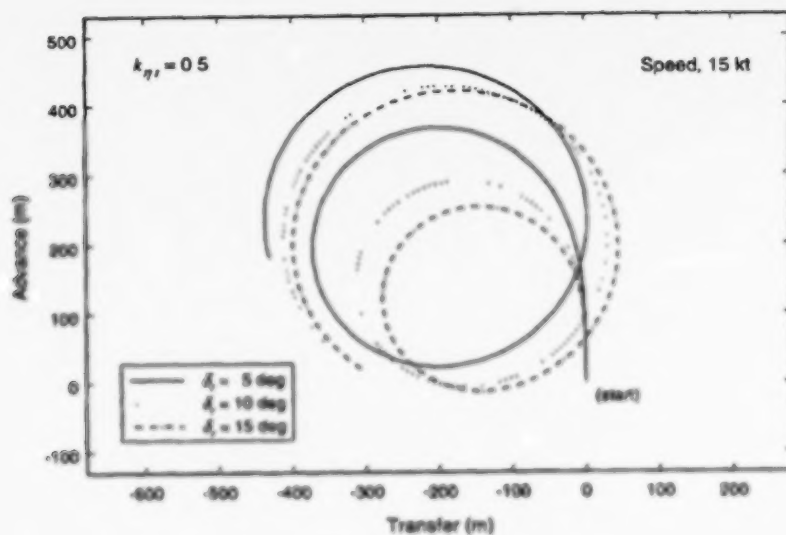


Figure 7. Horizontal pullouts for $k_{\eta r} = 0.5$ with $\delta_r = 5, 10$, and 15 degrees

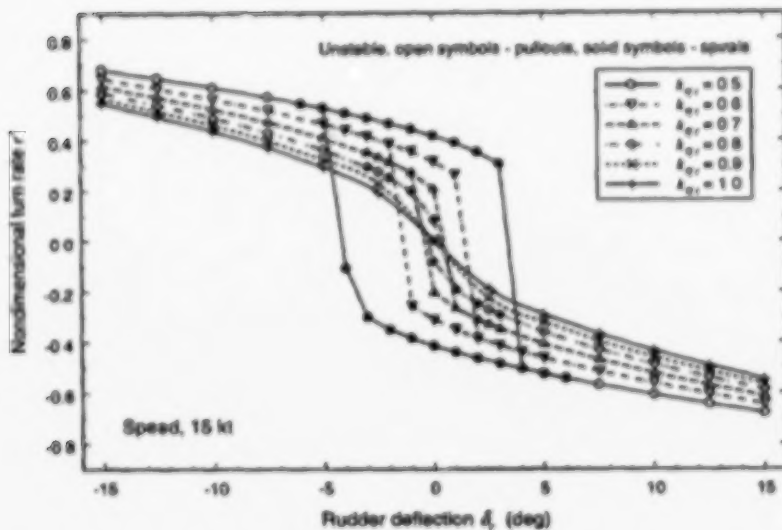


Figure 8. Nondimensional turning rate vs. rudder angle for $0.5 \leq k_{\eta r} \leq 1.0$

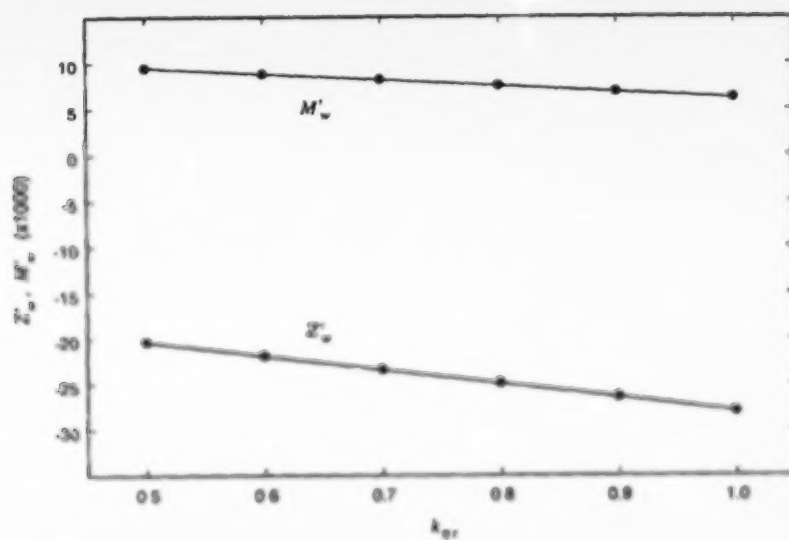


Figure 9. Pitch stability derivatives as functions of $k_{\eta I}$

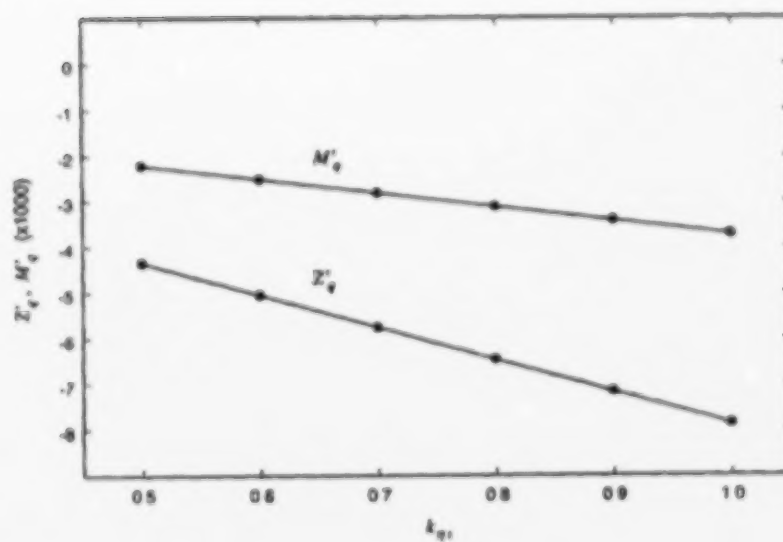


Figure 10. Pitch rate stability derivatives as functions of $k_{\eta I}$

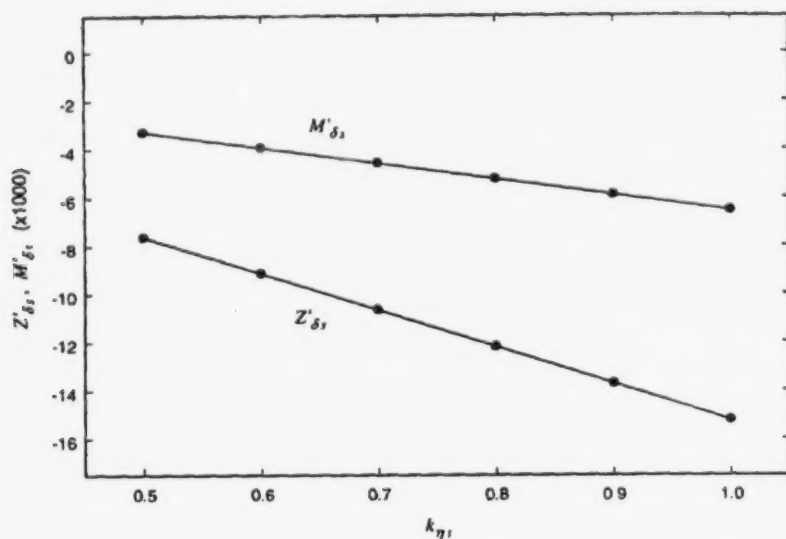


Figure 11. Sternplane deflection stability derivatives as functions of $k_{\eta t}$

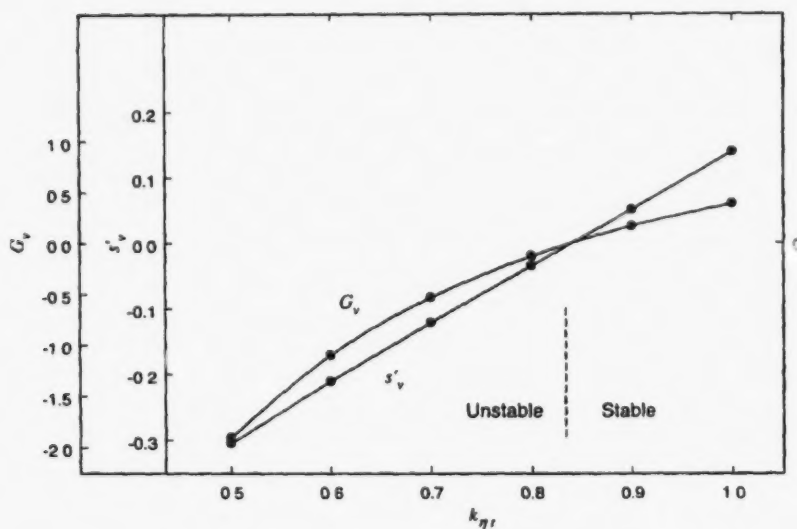


Figure 12. High speed vertical plane stability indicators as functions of $k_{\eta t}$

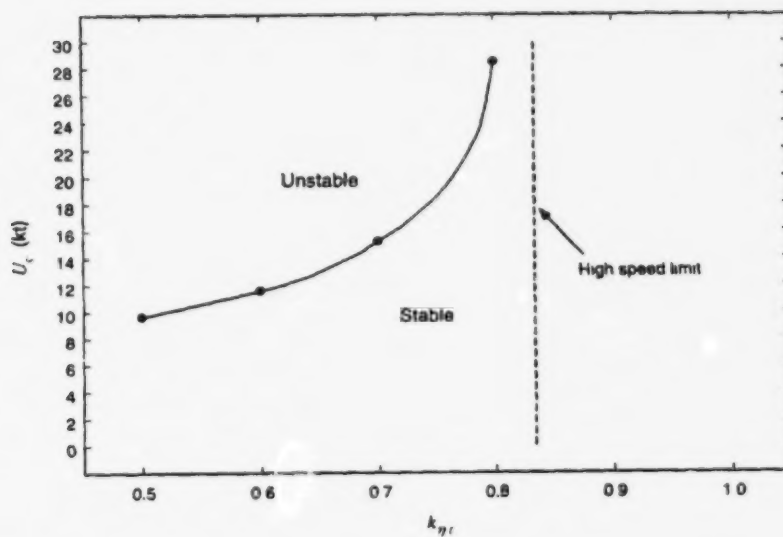


Figure 13. Vertical plane critical speed as a function of $k_{\eta t}$

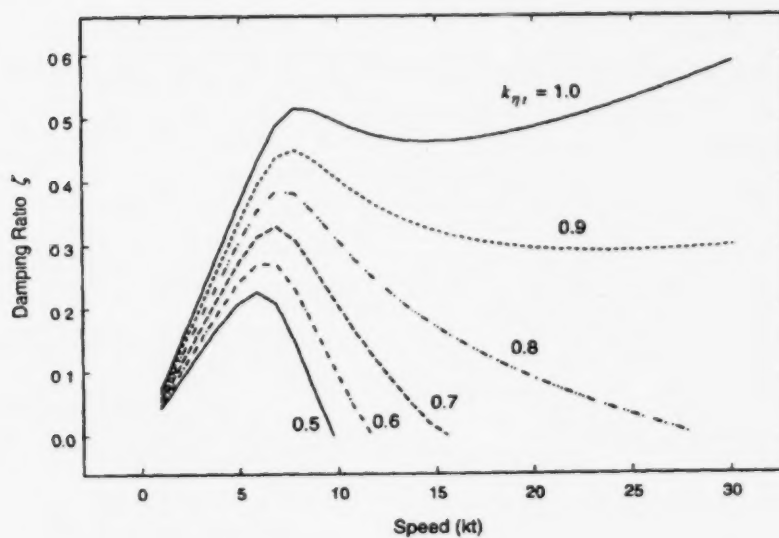


Figure 14. Ratio of actual to critical damping in the vertical plane as a function of speed

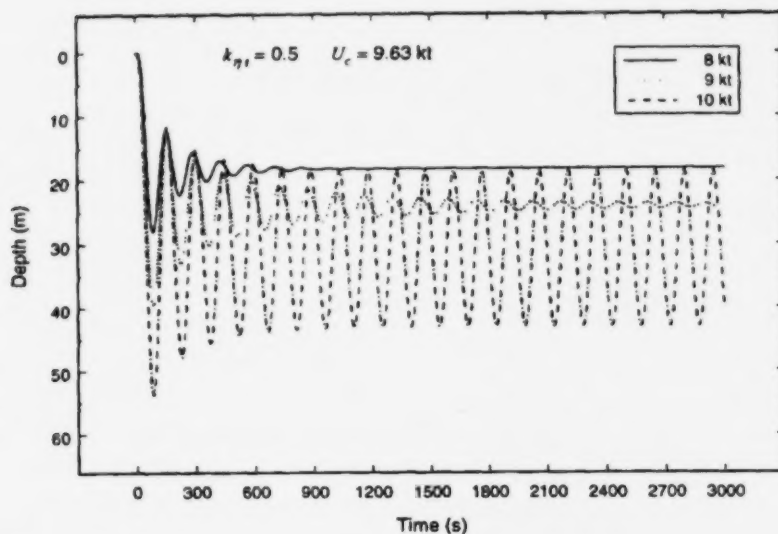


Figure 15. $10^\circ/10^\circ$ depth change for $k_{\eta t} = 0.5$; depth time history

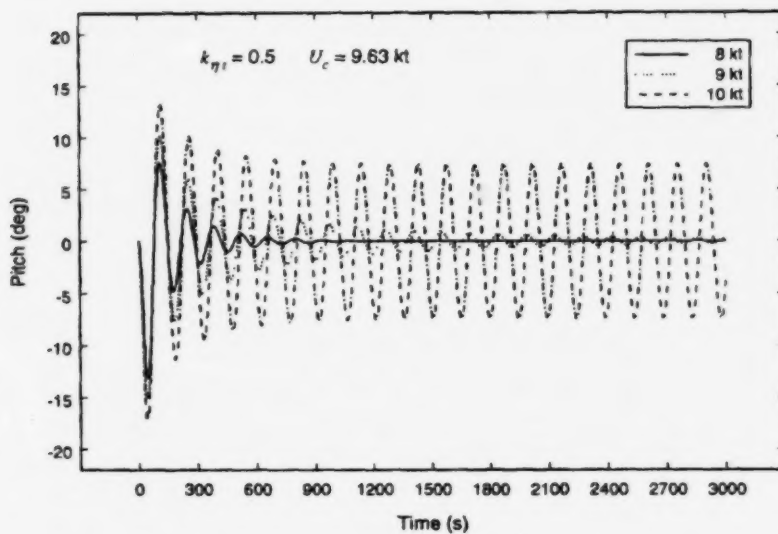


Figure 16. $10^\circ/10^\circ$ depth change for $k_{\eta t} = 0.5$; pitch time history

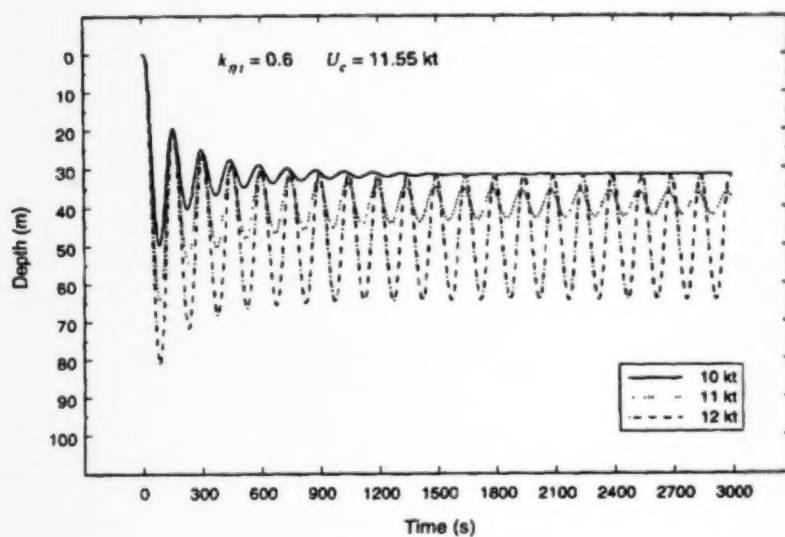


Figure 17. $10^\circ/10^\circ$ depth change for $k_{\eta t} = 0.6$; depth time history

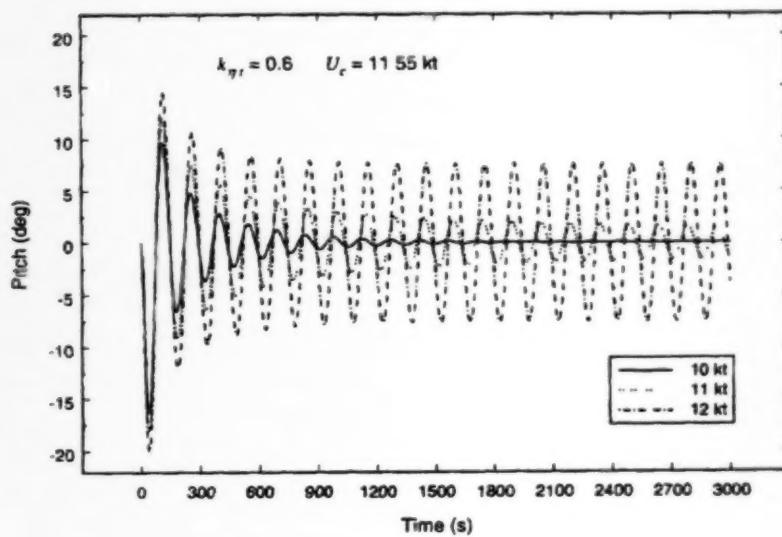


Figure 18. $10^\circ/10^\circ$ depth change for $k_{\eta t} = 0.6$; pitch time history

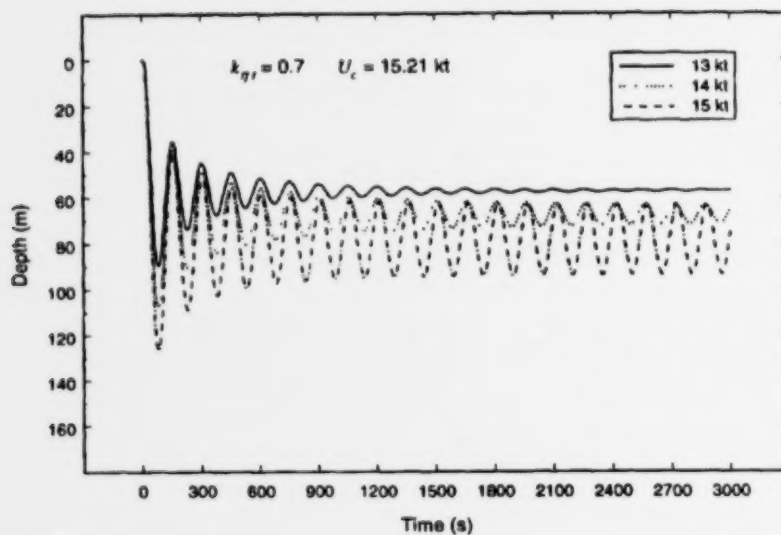


Figure 19. $10^\circ/10^\circ$ depth change for $k_{\eta t} = 0.7$; depth time history

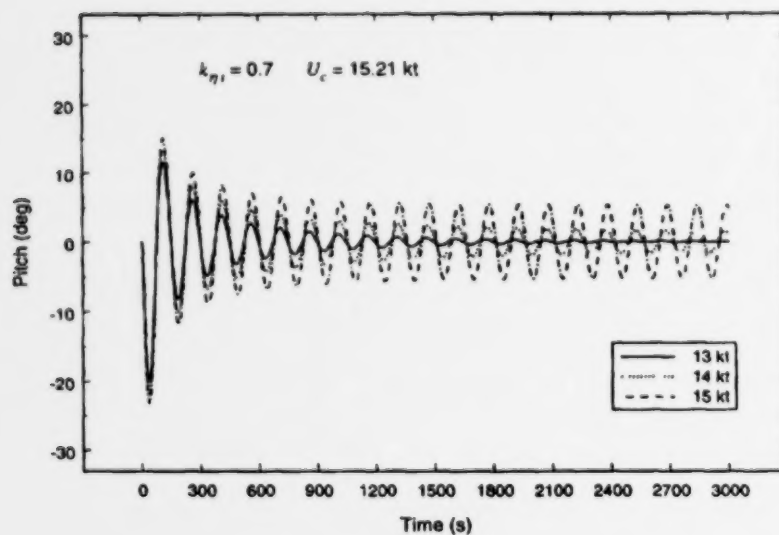


Figure 20. $10^\circ/10^\circ$ depth change for $k_{\eta t} = 0.7$; pitch time history

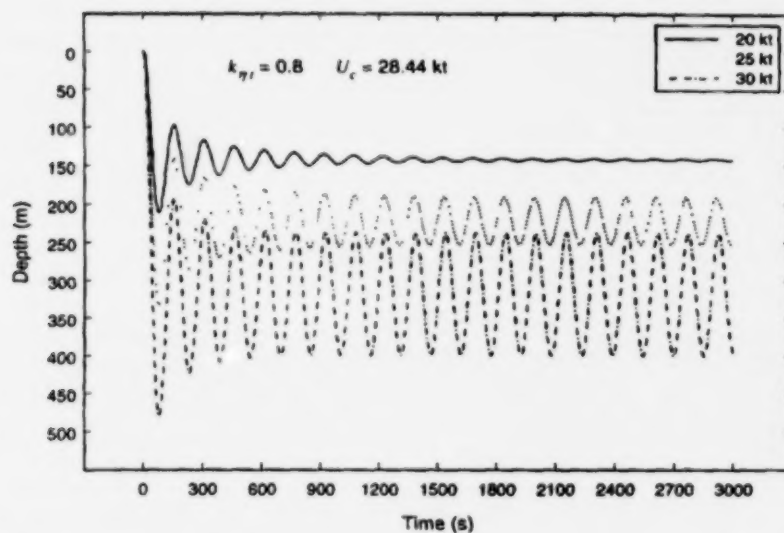


Figure 21. $10^\circ/10^\circ$ depth change for $k_{\eta t} = 0.8$; depth time history

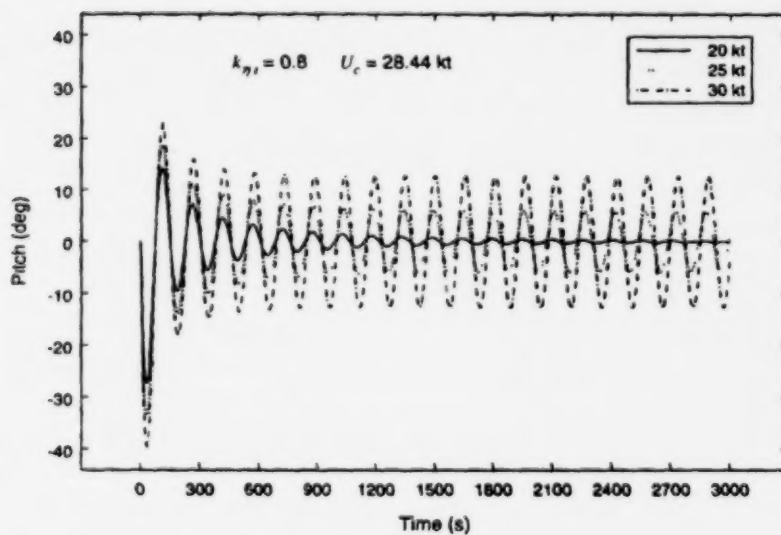


Figure 22. $10^\circ/10^\circ$ depth change for $k_{\eta t} = 0.8$; pitch time history

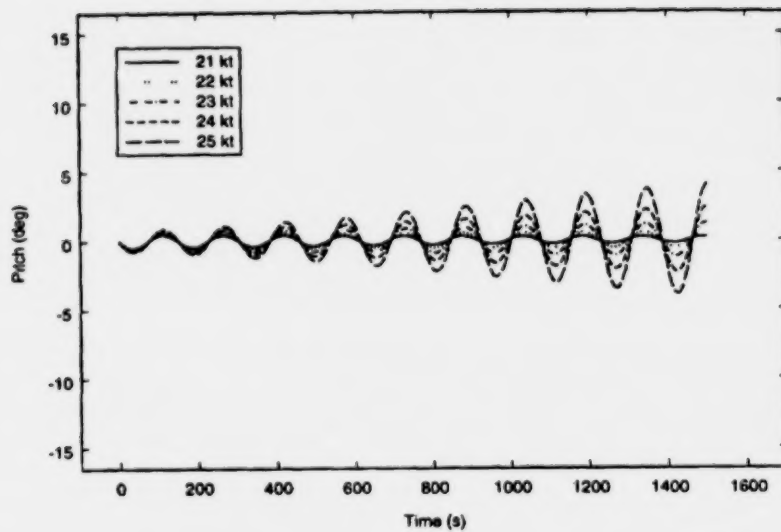


Figure 23. Pitch time history for $k_{\eta t} = 0.8$ following a small perturbation ($\delta_s = 1^\circ$ for 1 sec.); speed, 21 to 25 kt

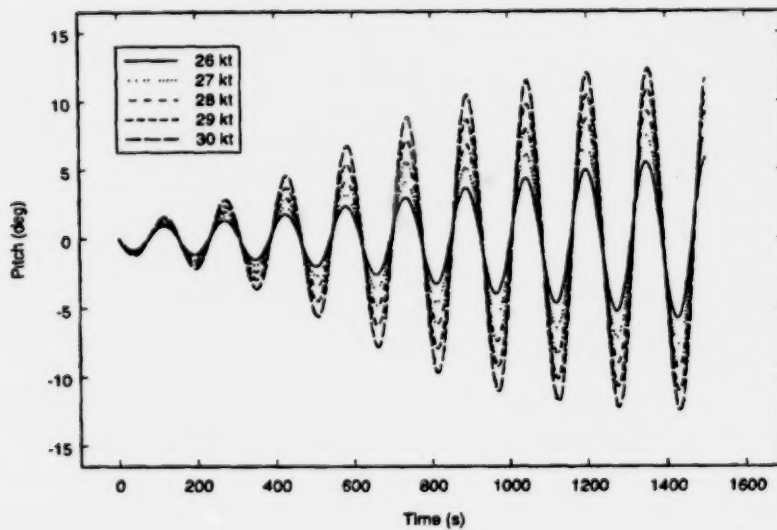


Figure 24. Pitch time history for $k_{\eta t} = 0.8$ following a small perturbation ($\delta_s = 1^\circ$ for 1 sec.); speed, 26 to 30 kt

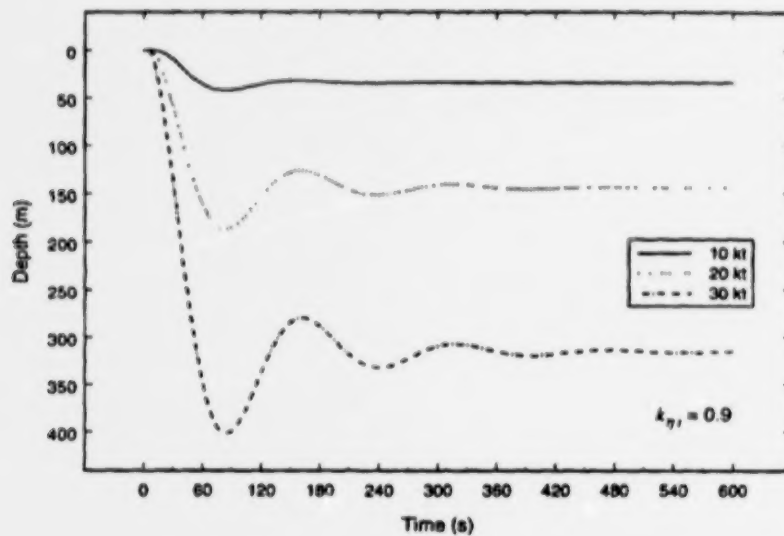


Figure 25. $10^\circ/10^\circ$ depth change for $k_{\eta t} = 0.9$; depth time history

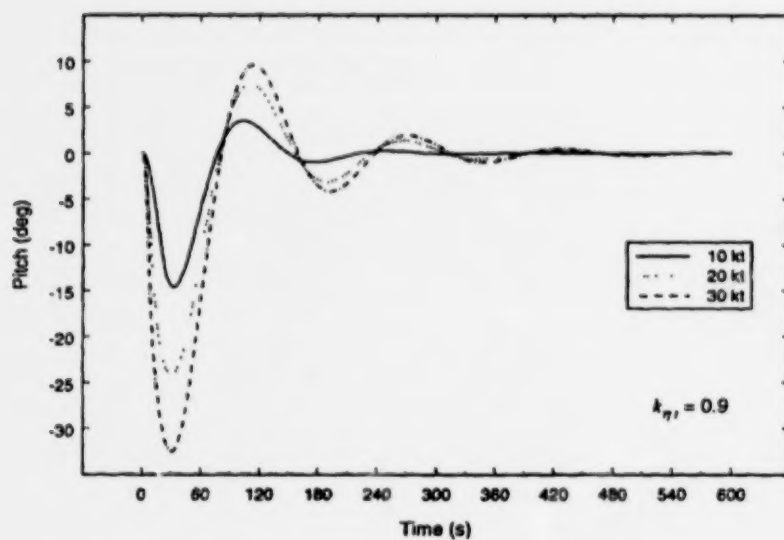


Figure 26. $10^\circ/10^\circ$ depth change for $k_{\eta t} = 0.9$; pitch time history

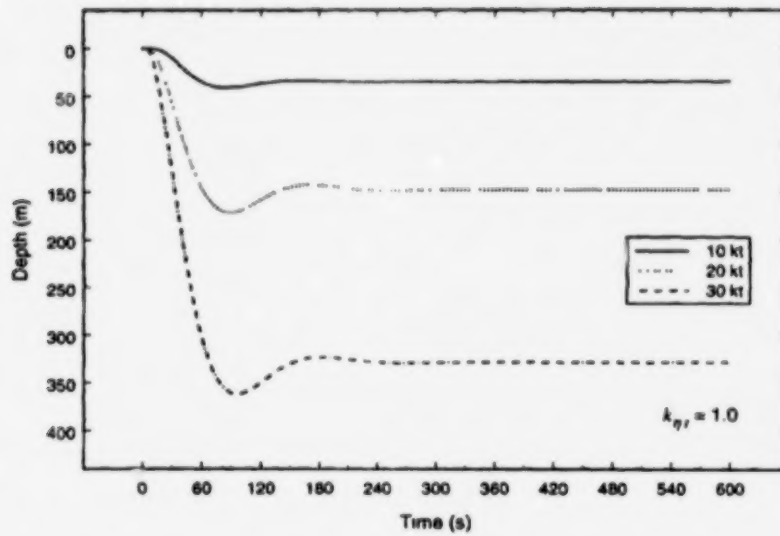


Figure 27. 10°/10° depth change for $k_{\eta t} = 1.0$; depth time history

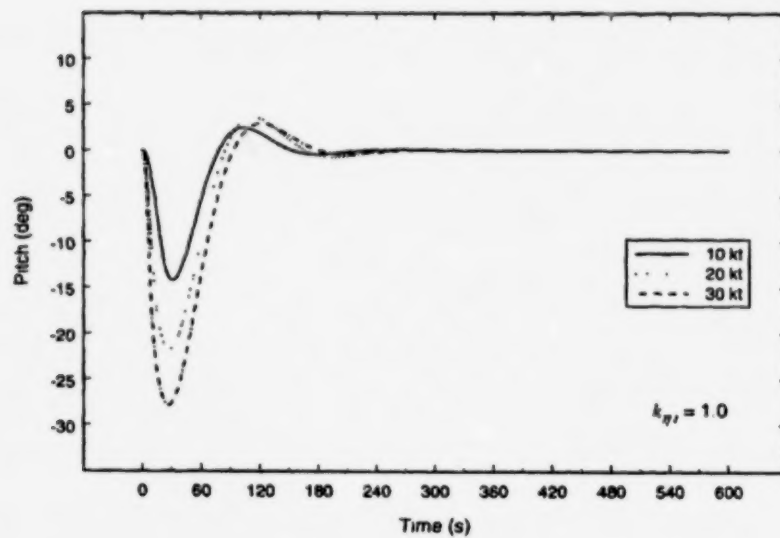


Figure 28. 10°/10° depth change for $k_{\eta t} = 1.0$; pitch time history

Annex A. Small-Perturbation Stability Analysis

This annex contains an outline of the analysis leading to stability indicators used in the body of this report. Standard hydrodynamic notation (see Nomenclature and reference [4]) is used except where noted. Only the principal results are presented here; for a more thorough description see references [9,10,11].

Horizontal Plane Stability

Motion is restricted to small perturbations v , $\dot{v}$, r , and $\dot{r}$ in the horizontal plane with controls fixed. The relevant linearized equations of motion are

$$m[\dot{v} + ur + x_G \dot{r}] = Y_{\dot{r}} \dot{r} + Y_v \dot{v} + Y_r r + Y_v v + Y_{\delta r} \delta_r \quad (\text{A.1})$$

$$I_z \dot{r} + m x_G [\dot{v} + ur] = N_r \dot{r} + N_v \dot{v} + N_r r + N_v v + N_{\delta r} \delta_r \quad (\text{A.2})$$

where u is the unperturbed axial velocity. In most cases, x_G is small (it is zero when CG is used as the longitudinal origin for the equations of motion), so terms containing it are often ignored; however, they are retained here for completeness.

Control is open loop, and it can be assumed without losing generality that $\delta_r = 0$. The characteristic equation of the resulting system is then

$$A\sigma^2 + B\sigma + C = 0$$

where

$$A = (m - Y_v)(I_z - N_r) - (N_v - m x_G)(Y_r - m x_G)$$

$$B = -[Y_v(I_z - N_r) + (m - Y_v)(N_r - m x_G) + N_v(Y_r - m x_G) + (N_v - m x_G)(Y_r - m u)]$$

$$C = Y_v(N_r - m x_G) - N_v(Y_r - m u)$$

The requirement for stability is that the real part of both roots (σ_1 , σ_2) of the quadratic be negative. From the Routh-Hurwitz criterion [12] this is equivalent to requiring

$$\frac{B}{A} > 0 \quad \text{and} \quad \frac{C}{A} > 0$$

Since, in practice, both A and B are positive [8], the remaining condition to be established for stability is $C > 0$.

This condition is generally expressed as the nondimensional stability index G_h [8] (note that $u' = 1$)

$$G_h = 1 - \frac{N'_v(Y'_r - m')}{Y'_v(N'_r - m'x'_G)} > 0 \quad (\text{A.3})$$

or as the nondimensional distance s'_h [11]

$$s'_h = x'_r - x'_v > 0 \quad (\text{A.4})$$

where

$$x'_r = \frac{N'_r - m'x'_G}{Y'_r - m'}$$

and

$$x'_v = \frac{N'_v}{Y'_v}$$

are nondimensional neutral points for the yaw rate (r) and yaw (v) derived forces respectively. For stability, x'_r must be forward of x'_v .

Now consider steady motion in the horizontal plane with the rudder deflected. Applying $\dot{v} = \dot{r} = 0$ and $\delta_r \neq 0$ to equations (A.1) and (A.2), eliminating v , and nondimensionalizing the result yields the slope of the curvature r' with respect to rudder angle as δ_r tends to zero

$$\frac{dr'}{d\delta_r} = \frac{Y'_v N'_{\delta r} - N'_v Y'_{\delta r}}{N'_v (Y'_r - m') - Y'_v (N'_r - m'x'_G)} \quad (\text{A.5})$$

The sign of this slope is negative for a stable submarine and positive for an unstable one as shown in figure 6. An analogous expression is obtained for $dv'/d\delta_r$ by eliminating r from equations (A.1) and (A.2).

Vertical Plane Stability

Motion is restricted to small perturbations w , $\dot{w}$, q , and $\dot{q}$ in the vertical plane with controls fixed. The relevant linearized equations of motion are

$$m[\dot{w} - uq - x_G \dot{q}] = Z_{\dot{q}} \dot{q} + Z_{\dot{w}} \dot{w} + Z_q q + Z_w w + Z_{\delta_s} \delta_s \quad (\text{A.6})$$

$$I_y \dot{q} - mx_G [\dot{w} - uq] = M_{\dot{q}} \dot{q} + M_{\dot{w}} \dot{w} + M_q q + M_w w + M_{\delta_s} \delta_s + M_{\theta} \theta \quad (\text{A.7})$$

These are directly analogous to equations (A.1) and (A.2) except that there is an additional term in equation (A.7) for the hydrostatic, or metacentric, restoring moment at pitch angle θ

$$M_{\theta} \theta = -mg \overline{BG} \theta$$

The term $M_{\theta} \theta$ is invariant with speed. As speed tends to zero, all other terms in the above equations vanish in order for the perturbations to remain small, and metacentric stability, i.e., $\overline{BG} > 0$, is the low speed stability criterion.

On the other hand, as speed gets large the hydrodynamic terms become dominant and $M_{\theta} \theta$ can be omitted, giving the equations of motion the same form as those for horizontal stability. It can be assumed that $\delta_s = 0$, and the characteristic equation of the resulting system is again quadratic

$$A\sigma^2 + B\sigma + C = 0$$

where now

$$\begin{aligned} A &= (m - Z_w)(I_y - M_q) - (M_w + mx_G)(Z_q + mx_G) \\ B &= -[Z_w(I_y - M_q) + (m - Z_w)(M_q - mu x_G) \\ &\quad + M_w(Z_q + mx_G) + (M_w + mx_G)(Z_q + mu)] \\ C &= Z_w(M_q - mu x_G) - M_w(Z_q + mu) \end{aligned}$$

As before, A and B are positive, so the condition for vertical stability at high speed is $C > 0$, which is conventionally expressed as

$$G_v = 1 - \frac{M'_w(Z'_q + m')}{Z'_w(M'_q - m'x'_G)} > 0 \quad (\text{A.8})$$

or

$$s'_v = x'_q - x'_w > 0 \quad (\text{A.9})$$

where

$$x'_q = -\frac{M'_q - m'x'_G}{Z'_q + m'}$$

and

$$x'_w = -\frac{M'_w}{Z'_w}$$

Given metacentric stability, equations (A.8) and (A.9) represent sufficient conditions for stability at all speeds. However, if the inequalities are not satisfied there is some speed at which the vertical plane dynamics change from stable to unstable.

Under the assumptions leading to equations (A.6) and (A.7), $q = \dot{\theta}$ and $\dot{q} = \ddot{\theta}$, so that (A.7) is second order in the perturbation θ . The characteristic equation of the resulting system is cubic

$$A\sigma^3 + B\sigma^2 + C\sigma + D = 0$$

where

$$\begin{aligned} A &= (m - Z_w)(I_y - M_q) - (M_w + mx_G)(Z_q + mx_G) \\ B &= -[Z_w(I_y - M_q) + (m - Z_w)(M_q - mu x_G) \\ &\quad + M_w(Z_q + mx_G) + (M_w + mx_G)(Z_q + mu)] \\ C &= Z_w(M_q - mu x_G) - M_w(Z_q + mu) - M_\theta(m - Z_w) \\ D &= Z_w M_\theta \end{aligned}$$

The three roots to the cubic comprise a complex conjugate pair, conventionally denoted σ_1 and σ_3 , and a real root σ_2 ; for stability, the real part of all three must be negative. The equivalent Routh-Hurwitz criterion is [12]

$$\frac{B}{A} > 0, \quad \frac{C}{A} > 0, \quad \frac{BC - AD}{B} > 0, \quad \text{and} \quad \frac{D}{A} > 0$$

Both A and B are positive, as is D when there is metacentric stability. Stability at a given speed is then determined by satisfying $BC > AD$, which also guarantees the additional condition $C > 0$.

Critical speed U_c for transition from stable to unstable behavior is obtained from the nondimensionalized equality $B'C' = A'D'$. Using

$$M'_\theta = -m' \frac{g\overline{BG}}{U_c^2}$$

and writing C'_R for the high speed asymptote of C' , i.e.

$$C' = C'_R - M'_\theta(m' - Z'_w)$$

U_c is given by

$$U_c = \sqrt{g\overline{BG} \frac{-m'(A'Z'_w + B'(m' - Z'_w))}{B'C'_R}} \quad (\text{A.10})$$

At intermediate speeds in the stable regime, the response to a perturbation is an underdamped oscillation, the characteristics of which are determined by the complex roots $\sigma_{1,3}$ of the characteristic equation. Palm [12] gives the general form of these roots as

$$\sigma_{1,3} = -\zeta\omega_n \pm i\omega_n\sqrt{1 - \zeta^2}$$

where ζ is the ratio of actual to critical damping and ω_n is the undamped natural frequency. It follows that

$$\zeta = \frac{-\text{Real}(\sigma_{1,3})}{|\sigma_{1,3}|} \quad \text{and} \quad \omega_n = |\sigma_{1,3}| \quad (\text{A.11})$$

Control Plane Reversal

A topic related to controllability, rather than inherent stability, in the vertical plane is control plane reversal. If a positive control force Z_δ is applied forward (typically, by a bowplane) of the heave neutral point x_w , the submarine will always pitch down and dive. However, if the point of application ($x_\delta = -M_\delta/Z_\delta$) is aft of x_w (as for sternplanes and, often, sailplanes), there is a plane reversal speed, $U_{c\delta}$, below which the submarine will dive while pitched up, and above which it will rise. At exactly $U_{c\delta}$ depth remains constant.

Consider the case of steady motion at $U_{c\delta}$ with pitch angle θ and control deflection δ ; substituting $\dot{w} = q = \dot{q} = 0$ into equations (A.6) and (A.7) gives

$$\begin{aligned} Z_w w + Z_\delta \delta &= 0 \\ M_w w + M_\delta \delta + M_\theta \theta &= 0 \end{aligned}$$

At a small pitch angle $\theta = w/U_{c\delta}$, eliminating δ from these equations yields the plane reversal speed

$$U_{c\delta} = \sqrt{gBG \frac{m}{\frac{1}{2}\rho L^2 Z'_w(x_\delta - x_w)}} \quad (\text{A.12})$$

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Annex B. DSSP20 Input Files for DARPA SUBOFF

DARPA SUBOFF geometry is given in reference [7]; the format of these files is described in reference [5].

Input for DSSP20A

" Case1010.AI

! The Casexxxx series is original 1998 Dsubf modified to fix k_etat on
! both rudders and sternplanes.

Title Case1010 - k_etat = 1.0 on rudders, 1.0 on sternplanes

Reference " See Roddy for reference location

-43.1991 .0 .0
.0 .0 .0

Hull

Label #Hull

Nose 0.0

Tail -91.44

Diameter

0	7.70915	9.23177
10.25121	10.82204	10.90194
10.90194	10.90194	10.90194
10.90194	10.90194	10.90194
10.90194	10.90194	10.90194
10.90194	10.66031	8.99180
6.08457	3.00293	1.28098

Lift " Sail

Label #Sail

Sail 5.45097

Planform

-19.83925 0. -5.45097 " l.e. root is actually at about -5.44303 ...
-19.83925 0. -9.86285
-27.74316 0. -9.86285
-27.74316 0. -5.45097
ToC 0.1810345

Lift " Tailplane - lower rudder position

Label #LowerRudder

Save

Tail 2.53030 5.45097

Planform

-81.59424 0. 3.28611
-82.72140 0. 5.45097

```
-85.99198 0. 5.45097
-85.99198 0. 1.77447
AllMoving
ToC 0.20
TailEff 1.0
```

```
Lift          " Tailplane - upper rudder position
Label #UpperRudder
Rotate 180
```

```
Lift          " Tailplane - starboard sternplane position
Label #StarboardSternplane
Save
Tail 2.53030 5.45097
Planform
-81.59424 3.28611 0.
-82.72140 5.45097 0.
-85.99198 5.45097 0.
-85.99198 1.77447 0.
AllMoving
ToC 0.20
TailEff 1.0
```

```
Lift          " Tailplane - port plane position
Label #PortSternplane
Rotate 180
```

```
Plot
! Maple
```

Input for DSSP20V

Title Casexxxx - Full scale D-Sub input for TailEff study

```
Prop
Location -91.44
Right Handed
RevLimit 250.0
RevRateLim 12.5
```

```
Lift 2
DelRateLim 7.0
Lift 3
DelRateLim 7.0
Lift 4
DelRateLim 7.0
Lift 5
DelRateLim 7.0
```

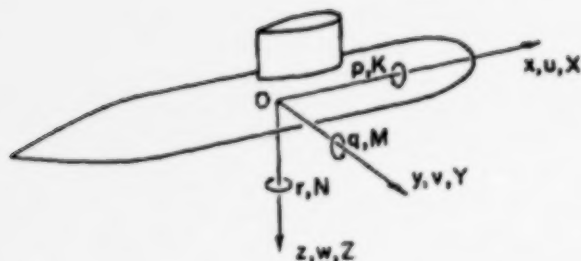
Nomenclature

Symbols

Many of the following symbols are used primed (') to indicate that they are nondimensional. Thus $x' = x/L$, $X' = X/\frac{1}{2}\rho L^2 U^2$, etc.; see reference [4].

b	tailplane semispan from the hull centerline
$\overline{BG}$	metacentric height
$\bar{c}$	mean appendage chord
g_r, g_s	factor for rotation-induced camber on the rudders or sternplanes
G_h, G_v	stability index for the horizontal or vertical plane
h_r, h_s	control effectiveness of the rudders or sternplanes
I_y, I_z	moments of inertia about the y and z axes
$k_{\eta t}$	tailplane (rudder or sternplane) efficiency
K, M, N	roll, pitch, and yaw moment
L	hull length
m	submarine mass
r_M	maximum hull radius
p, q, r	roll, pitch, and yaw rate
Re	Reynolds number, based on appendage mean chord or hull length
s_h, s_v	neutral point separation for the horizontal or vertical plane
u, v, w	axial, lateral, and normal velocity
U	speed
U_c	vertical plane critical speed
U_{cd}	sternplane reversal speed
x, y, z	vehicle body axes
x_G	axial coordinate of CG
X, Y, Z	axial, lateral, and normal force
Y_v, Z_w , etc.	coefficients in the equations of motion, see annex A
δ_r, δ_s	control deflection of the rudders or sternplanes
ζ	dynamic damping ratio
θ	pitch angle
ρ	density
σ	root of a characteristic equation
ω_n	undamped natural frequency

Coordinate System



Acronyms

BWB	Bundesamt für Wehrtechnik und Beschaffung — Federal Office for Defence Technology and Procurement (Germany)
CG	Center of Gravity
DARPA	Defense Advanced Research Projects Agency
DREA	Defence Research Establishment Atlantic
DSSP20	DREA Submarine Simulation Program, version 2.0

Unclassified

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11. DOCUMENT AVAILABILITY Unlimited		
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13. ABSTRACT Tailplane efficiency is a measure of the effectiveness of submarine tail appendages working in the hull boundary layer; it can have a significant impact on stability and manoeuvrability. This memorandum describes a study done to observe some of the effects for the DARPA SUBOFF configuration when tailplane efficiency is varied through a range of values that bracket the change from stable to unstable manoeuvring characteristics. The effects are essentially the same as would be obtained by varying the area of the sternplanes and rudders.		
14. KEYWORDS Rudders Sternplanes Tailplane Efficiency DARPA SUBOFF		

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